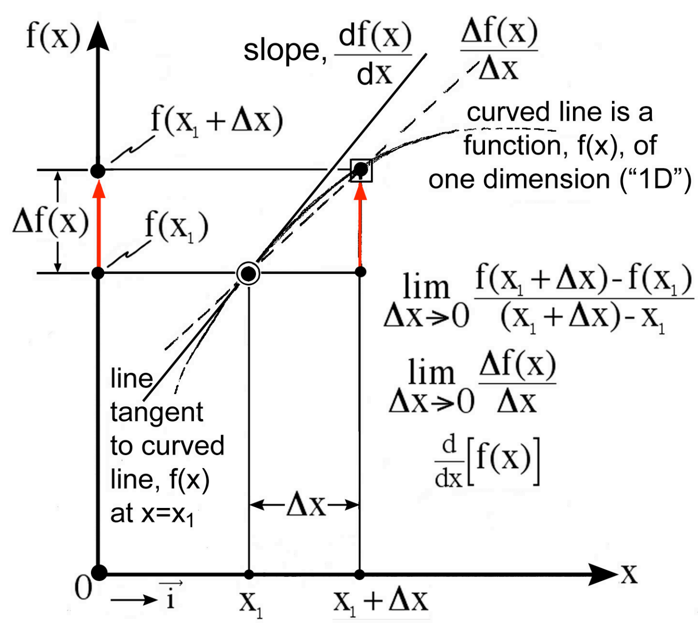
Three Visual Methods: Envisioning Gradients. Cognitive Visual Data Compression Method ©

Compress large graphical-tabular data sets into a visual 3D gradient format

Envisioning gradients requires a review of our fundamental concepts of a gradient. Gradients are associated with a change in a property. For example if a student was given a task of measuring temperature in a room and recorded that the temperature changed 5 degrees, the question is raised -- how did the temperature change? Did the temperature change with respect to a change in position (x,y,z) in the room or was the position held constant and this temperature changed with respect to time, t? It is implicity understood that gradients of properties are associated with a change in an independent variable, e.g. space or time. When continuous functions are used to describe this gradient we refer to the function by describing the shape as ... "it goes like" linear, exponential, etc. It is possible that properties could simultaneously change with respect to space (x,y,z) (three dimensions) (3D) and time, t, the fourth dimension (4D). The idea that any property (scalar, vector, or tensor) can change with respect to space and time is fully developed in continuum mechanics as a comoving derivative. The discussion here focuses on envisioning gradients of scalar properties in one dimension (1D), two dimensions (2D), and three dimensions (3D).

Visual gradients in 1D: envisioned as curved lines

Our idea of a gradient in one dimension (x) requires us to think in two dimensions where the function, f(x), is shown as a curved line in a two dimensional plane, Fig. 1. The scalar function, f(x), is drawn along the vertical axis as an dependent variable and the independent variable, x, is drawn in the horizontal direction which creates a two dimensional plane. Here the gradient of the scalar function, f(x), changes only with respect to the x-axis. This increase is graphically shown by the red arrow shown in Fig. 1. The idea of a change in the property, function f(x), with respect to a corresponding change in the independent variable, x, is fundamental to our concept of a gradient. The mathematical statement of this same idea in the limit is the definition of the derivative which is shown as a line tangent to the curved line in Fig. 1.

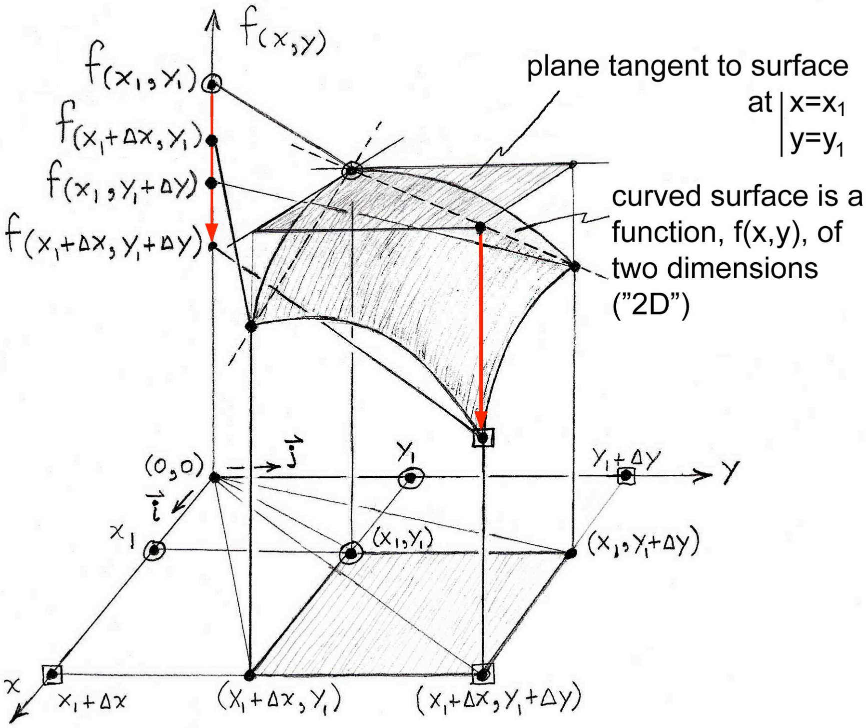


$$\overrightarrow{\text{grad}}\, f(x) = \frac{d}{dx}\left[f(x)\right]\ \vec{i}$$

Figure 1. Mathematical definition and graphical representation of one dimensional scalar functions and their gradients envisioned in a two dimensional plane.

Visual gradients in 2D: envisioned as curved sufaces

Similarly our idea of a gradient in two dimensions, x and y, requires us to think in three dimensions where the function, f(x,y) is shown as a curved surface in Fig. 2. The scalar function, f(x,y), is drawn along the vertical axis as a dependent variable and the two independent variables (x,y) are drawn as two axis both perpendicular to f(x,y) axis and also perpendicular to each other. Here the function, f(x,y) is observed to decrease both along the x-axis and the y-axis independently which creates a plane tangent to the curved surface and this change is graphically represented by the red arrow pointing downward in Fig. 2. The mathematical equivalence to this tangent plane is the equation for a gradient of the function, f(x,y), which is shown in the bottom portion of Fig. 2. NOTE!!! the curved surface, although drawn in three dimensions, is a two dimensional function. There is no z-axis shown here as a third independent variable, but rather the vertical axis is replaced with a dependent function, f(x,y), in Fig. 2. All to often these raised curved surfaces are referred to as three dimensional functions, because they are envisioned in three dimensional space.

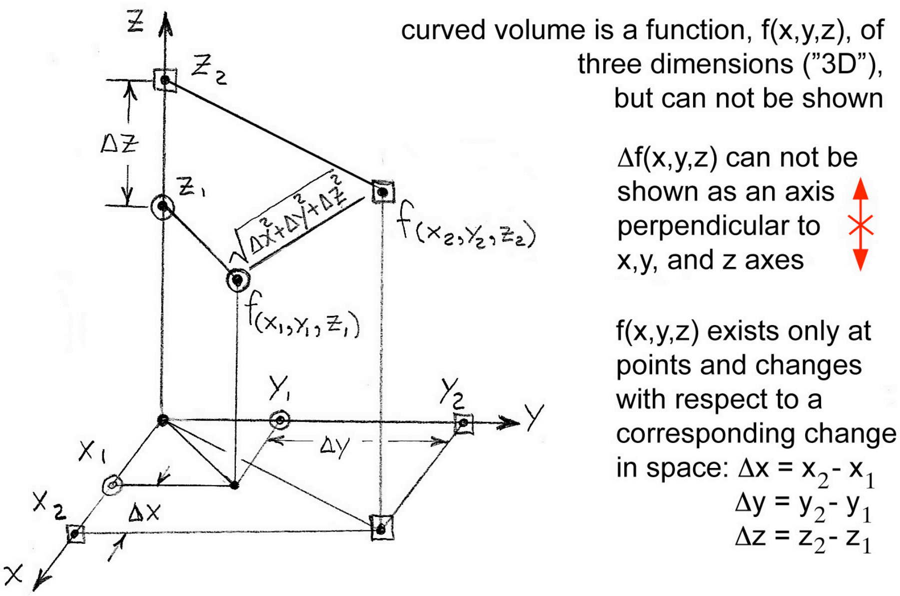


$$\overrightarrow{\text{grad}}\, f(x,y) = \frac{\partial f(x,y)}{\partial x}\bigg|_{y=y_1} \vec{i} + \frac{\partial f(x,y)}{\partial y}\bigg|_{x=x_1} \vec{j}$$

Figure 2. Mathematical definition and graphical representation of two dimensional scalar functions and their gradients envisioned in three dimensional space.

Visual Gradients in 3D: envisioned as curved volumes

Our idea of a gradient in three dimensions (x, y, and z) would require us to think in four dimensions where taking the gradient of the function, f(x,y,z) would be represented by a curved volume. However drawing a geometric shape of a three dimensional function, f(x,y,z), for all possible values of x, y, and z is not possible. This would also require that a fourth axis for f(x,y,z) be drawn perpendicular to the x, y, and z axes in Fig. 3, which is also not possible. However the gradient of a scalar funtion, f(x,y,z), can be evaluated at a point (x_1,y_1,z_1) in the limit as Δx, Δy, and Δz go to zero. It is also possible to visually represent a **single** value of the function, f(x,y,z) as a collection of points near (x_1,y_1,z_1) that would map out a curved isosurface in 3D space. This method will be discussed in the next section, Envisioning Pattern Function Extraction.



$$\overrightarrow{\text{grad}}\, f(x,y,z) = \left.\frac{\partial f(x,y)}{\partial x}\right|_{\substack{y=y_1\\ z=z_1}} \vec{i} + \left.\frac{\partial f(x,y,z)}{\partial y}\right|_{\substack{x=x_1\\ z=z_1}} \vec{j} + \left.\frac{\partial f(x,y,z)}{\partial z}\right|_{\substack{x=x_1\\ y=y_1}} \vec{k}$$

Figure 3. Mathematical definition and graphical representation of three dimensional scalar functions and their gradients envisioned in three dimensional space. Envisioning three dimensional gradients as curved volumes is not possible. However the gradient of a scalar funtion, f(x,y,z), can be evaluated at a point (x_1,y_1,z_1) mathematically.

It is possible however to draw gradients for three dimensional functions, f(x,y,z), using orthogonal intersecting planes constructed from small multiples, Tufte [1], of a family of one dimensional functions, f(x), shown in Fig. 1. Compression of this function, f(x), is extended to include a family of curves with respect to a second independent variable, f(x,T). This method is developed in the next section where all of these 1D functions are compressed into three planes and observed as color gradients. Drawing functions as color gradients is not new. However the conceptual link of color gradients to a compressions of 1D functions using Tufte's small multiple idea provides an insightful method of describing gradients in 3D. If three orthogonal planes intersect at a common point, e.g. (x_1,y_1,z_1) and move in the neighborhood of this point (possibly oscillate) along their respective axes normal to each plane, gradients in 3D can be envisioned using this idea of a compressed 1D format. Because this matches our intuitive (mathematical) concept of a 3D gradient, this compression scheme is refered to as a cognitive visual data compression (CVDC) method.

Conversion of 1D family-of-curves into visual 3D gradients
(Development of the Cognitive Visual Data Compression (CVDC) Method)

This method requires a uniformly distributed data set. This requirement is not unusual with the advent of supercomputers and computer automated laboratory equipment. With such equipment results can be constructed in uniformly spaced arrays where no data are eliminated simply because generating these data is no longer tedious. For example in computer tomography systems such detail can become a life saving requirement. But here we create our uniform mesh from a complex function. Such complex functions are common in dynamic systems. To continue the discussion a function of three independent variables (x,T,t) is created:

$$F(x,T,t) = \frac{\ln\left[P_1(30-T) + P_2(1-A)^2\right]}{2.5\,P_2(1-A)^2}$$

$$\text{where } A = \frac{5}{4}\sin(t\,L_1\,\pi / 180)\sin(T\,L_2\,\pi / 180)\sin(x\,L_3\,\pi / 180)$$

$$P_2 = e^{\{TP_1 + \frac{1}{2}\sin(x\,L_1\,\pi / 180)\}}$$

$$P_1 = 0.6\left(\frac{x+1}{360}\right)^2 + 0.05\left(\frac{x+1}{360}\right)$$

$$L_1 = t/4,\ L_2 = T/3,\ L_3 = x/45,\ 1 \le t \le 30,\ 1 \le T \le 30,\ 0 \le x \le 360 \quad (1)$$

which can be shown to approach zero in the limit as T approaches 30. Typically these functions are shown as a family of curves. We are very comfortable with this format because we are first taught to imagine functions as shapes (curved lines or surfaces), Figs. 1 and 2. A line tangent to a curve is the derivative and the area under that same curve is the integral of the function. Let's convert a 1D function from a family-of-curves format into a compressed parametric space. We start by arbitrarily selecting four figures, as shown below in Fig. 4, what Tufte [1] would call small multiples. Each figure represents a function F(x,T) at a single instant in time, t, that contains a family of curves each at a different time period, T. Because T approaches zero in the limit as T approaches 30 only a few curves are shown together with the 27th curve which is observed to be near zero as expected. Except for such limiting trends it is difficult to understand how this function behaves until we draw at least a few curves as shown below. With this format our view is limited to a collection of about four to thirty figures total, because this represents a range of figures that can fit on a standard printed page. These same small multiples of thirty figures can be viewed as an animation. Below we show a minimum of four figures.

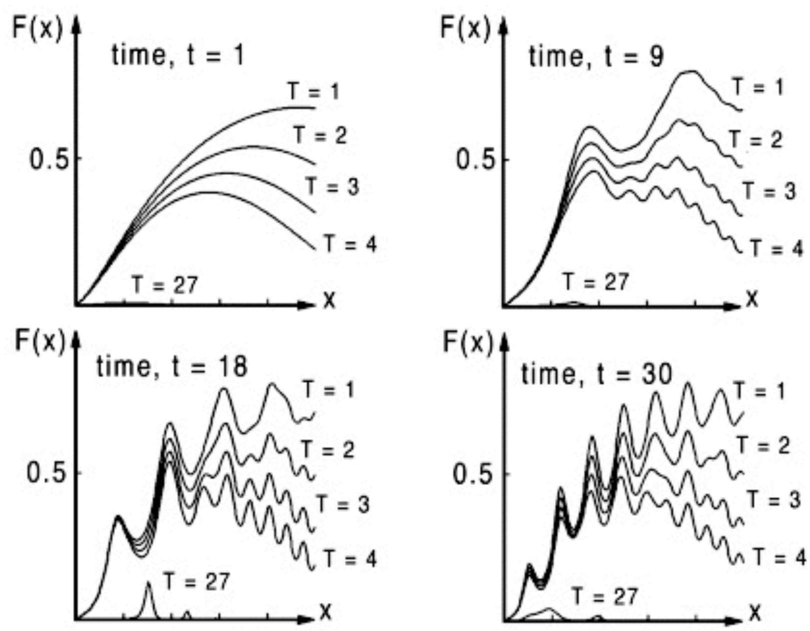


Figure 4. Small multiple of F(x,T,t) where only five curves are shown in each figure, since this function tends to zero as T approaches 30. For comparison a total of 30 figures can viewed as an upper limit that can fit on a standard printed page.

Such complex functions are common to many dynamic systems. When solutions are "chaotic" we have an excellent example of how even simple graphs assist us in understanding functions by observing patterns that would first appear to be totally random or chaotic. This complexity motivates us to view a larger number of curves in each figure.

The next figure, Fig. 5, demonstrates that a family of 30 curves in each figure not only represents an visual limit in our ability to view data in this format but reveals additional information assumed absent in the previous figure, Fig. 4. In fact we might assume that such an instability would most likely occur in the upper right region where there appears to be more variation with time. However the largest gradient in F(x,T,t) is unexpectedly observed in the lower left region at time, t=18 and disappears at later times in Fig. 5. This can be more clearly seen if all 30 figures at each time step are compared on one page. These same small multiples of thirty figures can be viewed as an animation.

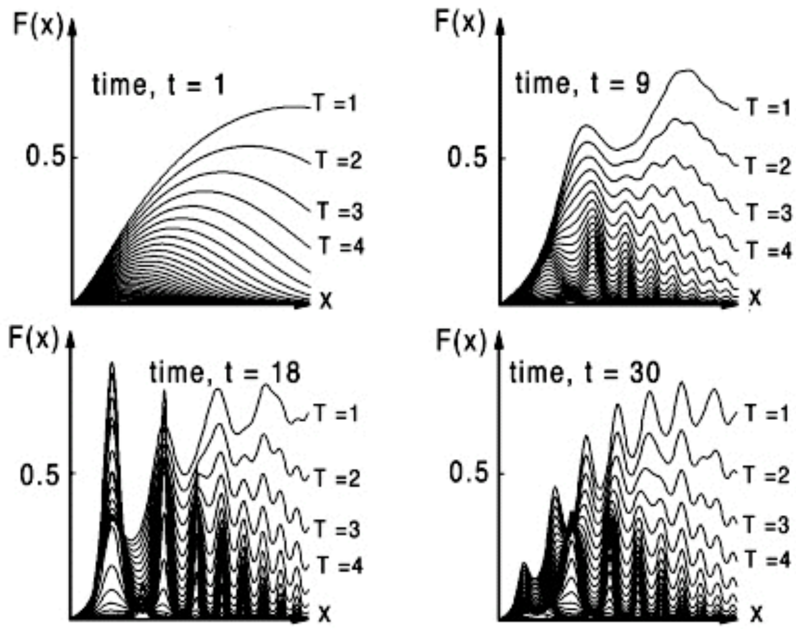


Figure 5. Small multiple of F(x,T,t) where an instability occurs unexpectedly near t=18 as t approaches 30. This gradient in time is more clearly seen if all thirty figures for each time step are plotted on one page.

Let's say that for the function F(x,T,t) we are interested in observing a total of 300 time steps (300 figures) with 300 time periods, T, curves in each figure. This would be a total of 90,000 1D curves. Obviously this would result in an incomprehensible blur of curves in each figure. Such a format would be useless to the observer.

A typical approach to this problem would be to determine a priori what information to present and what information to avoid. This process of eliminating irrelevant data is a tedious, if not inaccurate, process where we typically assume trends in complex analytic functions, experimental, or numerical simulation data. Each individual has their own unique, if not ambiguous, way of doing this. This process of simplifying is also motivated in order to organize data for presentation. To avoid this ambiguity we propose to compress all 90,000 curves into a single space without eliminating any data. Such an approach, if possible, transcends using graphics for presentation and enables the observer to use graphics for analysis and discovery. This method would also allow insightful comparisons within the compressed format. Can this be done in general?

We start by arbitrarily taking the 8th and 30th time slice from the small multiple figure. We start here because a family of 30 curves in a single figure is obviously getting "too busy" and we believe this presentation format represents an upper limit. But we want more than 30 figures, let's say n-figures, where each figure contains a family of m-curves.

We proceed first by compressing the 30 curves in Fig. 5 at t=8 and show that this method can be extended to m=300. We start this compression process by placing a color bar next to the left most figure shown in

Fig. 6. Next we pick an arbitrary value of x and find the corresponding value of the function F(x). But we continue moving to the left and observe the corresponding color for F(x). We take that color and map it back onto the line. We continue this coloring process by mapping color onto the entire line. Now the color along the line and the shape of the line contain redundant information, hence we can remove the shape but keep the color and move the colored flat line vertically downward to the figure below without losing information. Obviously we can continue this process and easily move all 30 curves into the same space without confusion. This method should also work for 300 lines. It would not be possible to show the same information as 300 shapes in the traditional family-of-curves format.

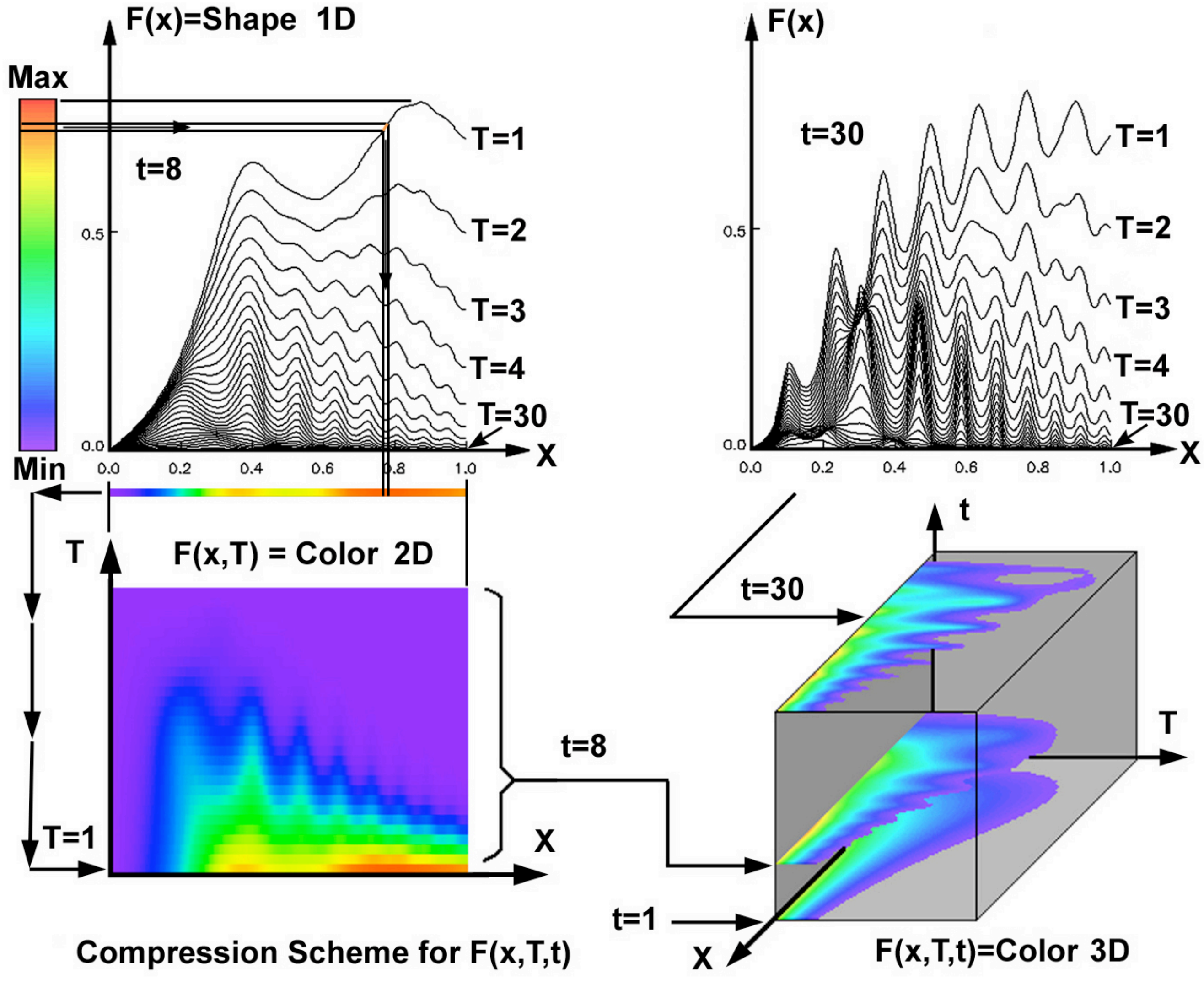


Figure 6. Compression Scheme for F(x,T,t)

Functions compressed into such formats are not new. We often see these "color plots" in literature and give little thought to why we find these figures useful nor are most of us aware of their functional relationship to a 1D family-of-curves format. This functional equivalence of color-shape-function allows us to visually generalize the functional visual method even further. For the same reason we can compress m-curves into a 2-D plane, we can also compress n-figures into a 3-D structure by vertically stacking each figure as a 2-D plane shown at the lower right of Fig. 6. It is now reasonable to claim that if there are "l" points for x, "m" points for T, and "n" points for t, then we can compress (l x m x n) pieces of data into a compressed visual format that is superior to the traditional 1D family-of-curves format. We now select a visual tool that will allow us to sort through this compressed visual format. This can be simply accomplished by interactively moving the horizontal colored x-T plane, shown at the lower right of Fig. 6, vertically upward with increasing time and stopping at any arbitrary time for comparison. In Fig. 6 we captured the 1st, 8th, and 30th time plots where the lower values (purple) have been erased in each plane to aid the observer in comparing how the pattern, hence the function, changes with time. Although useful, this method nevertheless reveals little new information about our complex function. Again we return to using graphics to PRESENT what we already functionally understand. In this case our images simply represent an alternate interesting way to present many 1D functions into a compressed space but perhaps not very revealing.

However if we choose the vertical x-t plane and move this colored plane from the left to the right and observe how this function changes as shown in Fig. 7. Obviously these new colored patterns represent the same function but now the pattern is drawn without prior thought. For most people this new image creates an insightful if not revealing experience. After experimenting with several investigators using this method on a variety of data sets (analytic, experimental, and numerical), the response was very interesting. In all cases this function was not thought of before (a priori) and typically the investigator enthusiastically proceeded with this new method operating on other functions or data sets. What appears to be new information in the other planes shown in Fig. 7 is the visual equivalent to our idea of a gradient of a scalar function. In its simplest form such a gradient can be constructed as a 1D quantity, where we fix a value for T and t and proceed to track how F(x,T,t) changes along a line parallel to the x-axis. But in Fig. 7 we observe gradients for an entire range of T and t. Unlike Figs. 1 and 2, it is not necessary to mathematically specify a single point in each plane, since the gradient for each of the three planes applies for all points in that plane. When all three planes are combined, gradients in three dimensions (3D) have been approximated from a collection ("compression") of numerous 1D curved lines. Stacked planes in Fig. 7 approximate color gradient changes corresponding to movement. Hence our idea of a gradient in 1D extends graphically to representing gradients in 3D. Note, this technique can be used for any arbitrary function.

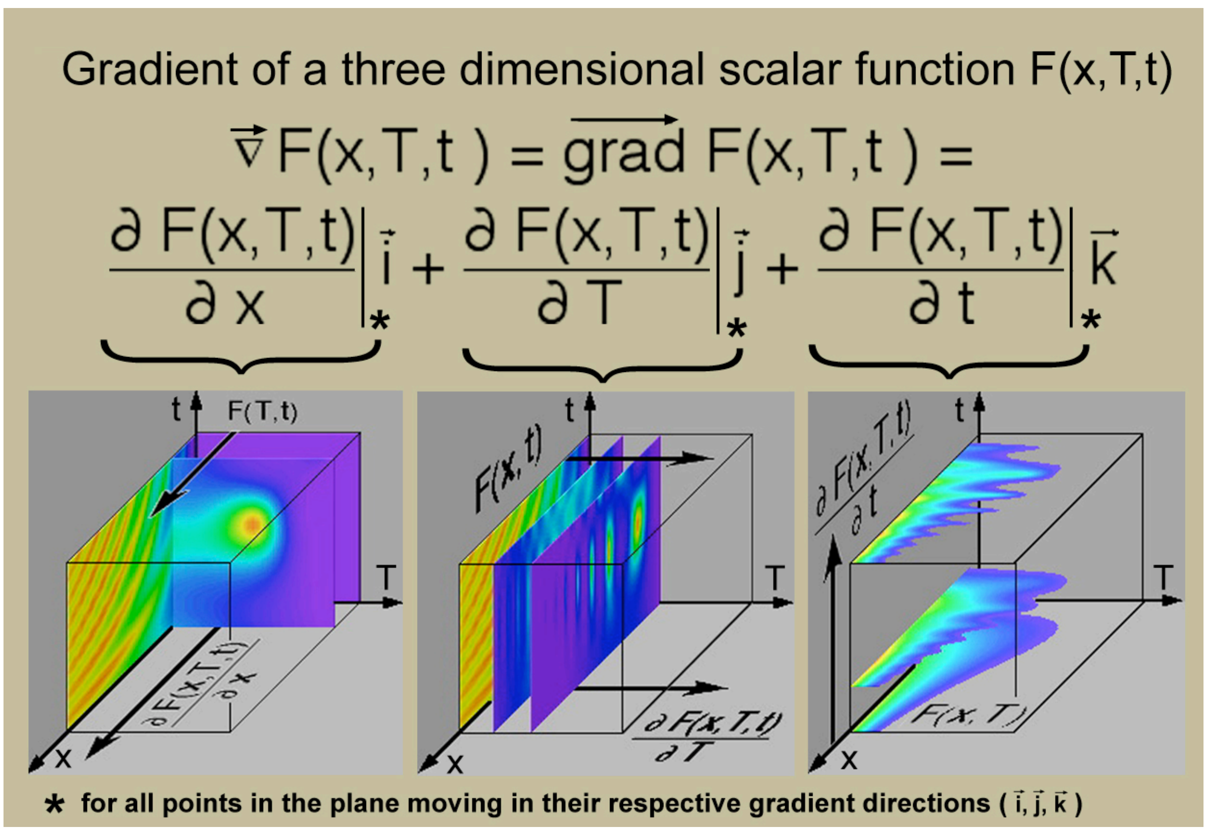


Figure 7. 3D gradient of a scalar function visualized when color patterns change with moving orthogonal planes.

Further investigation reveals that uncompressing the x-t xplane would be equivalent to redrawing 300 figures with 300 different curves in each figure where t and T are interchanged within the traditional 1D family-of-curves format. This TEDIOUS task of redrawing 90,000 curves reduces to a much simpler (if not revealing) visual method of moving and observing changing patterns <-> functions as we move the x-t plane through various values for T.

Cognitive Patterns

The reason why this visual method is revealing in most investigations can now be simply explained by the same cognitive mechanism described by Richard Friedhoff's explanation of visual experiments [2] conducted by psychologist Donald Michie. Namely, most people tend to exchange arithmetic calculations (functions) with visual patterns. An excellent example is to show all possible combinations of three integers (1,2,3,4,5,6,7,8,9) that add to 15 without repeating integers, Fig. 8. What could again be a TEDIOUS task is made simple with the following pattern, but try this exercise first without the pattern.

15	15	15	15	15
15	6	7	2	15
15	1	5	9	15
15	8	3	4	15
15	15	15	15	15

Functional Pattern

X	O	O
X	O	X
O	X	X

X	O	O
O	X	X
O	X	O

Pattern Only (arbitrary)

Figure 8. Simple Cognitive Patterns

In some cases adding color to such functional patterns aids us in this cognitive intuitive process of understanding the origin of functions (i.e. fractals). Interestingly this same functional pattern of numbers can be reduced to a simple tic-tac-toe pattern, Fig 8.

The important point is that we all tend to exchange complex if not tedious functional operations with patterns. With these patterns we can make comparisons that would otherwise go undetected even if these functions are written in their traditional mathematical script (equations), simple graphs, or tabular format. This same point is made by Edward Tufte[1] when he describes William Palyfair's efforts (1759 -1823) to complement (not replace) functions shown in tabular format with a more comparative set of graphs. Comparison of even simple functions in graphical format is cognitively superior to the tabular format but lacks the accuracy. This is demonstrated by Tufte in Fig. 9 below.

I		II		III		IV	
X	Y	X	Y	X	Y	X	Y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

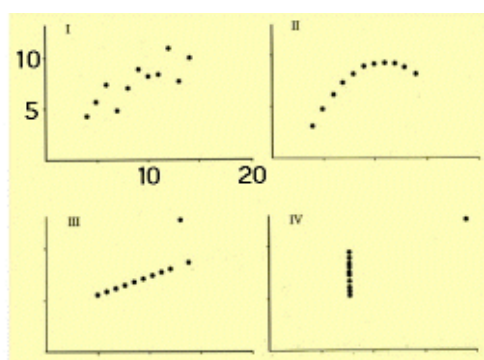


Figure 9. From E.R. Tufte [1], "The Visual Display of Quantitative Information."

This emphasis on accuracy and precision kept simple graphs from scientific archival journals for some 150 years, although comments have been made that this latency in publishing graphics could have been influenced more by the printing technology of the times. Obviously scientists and engineers routinely use both today when appropriate. Perhaps our recent (1985 - 2005) improvements in graphic technology has prompted yet other choices in how we explore and present our complex functions and massive data sets.

This chronology of envisioning scientific information is summarized in Fig. 10.

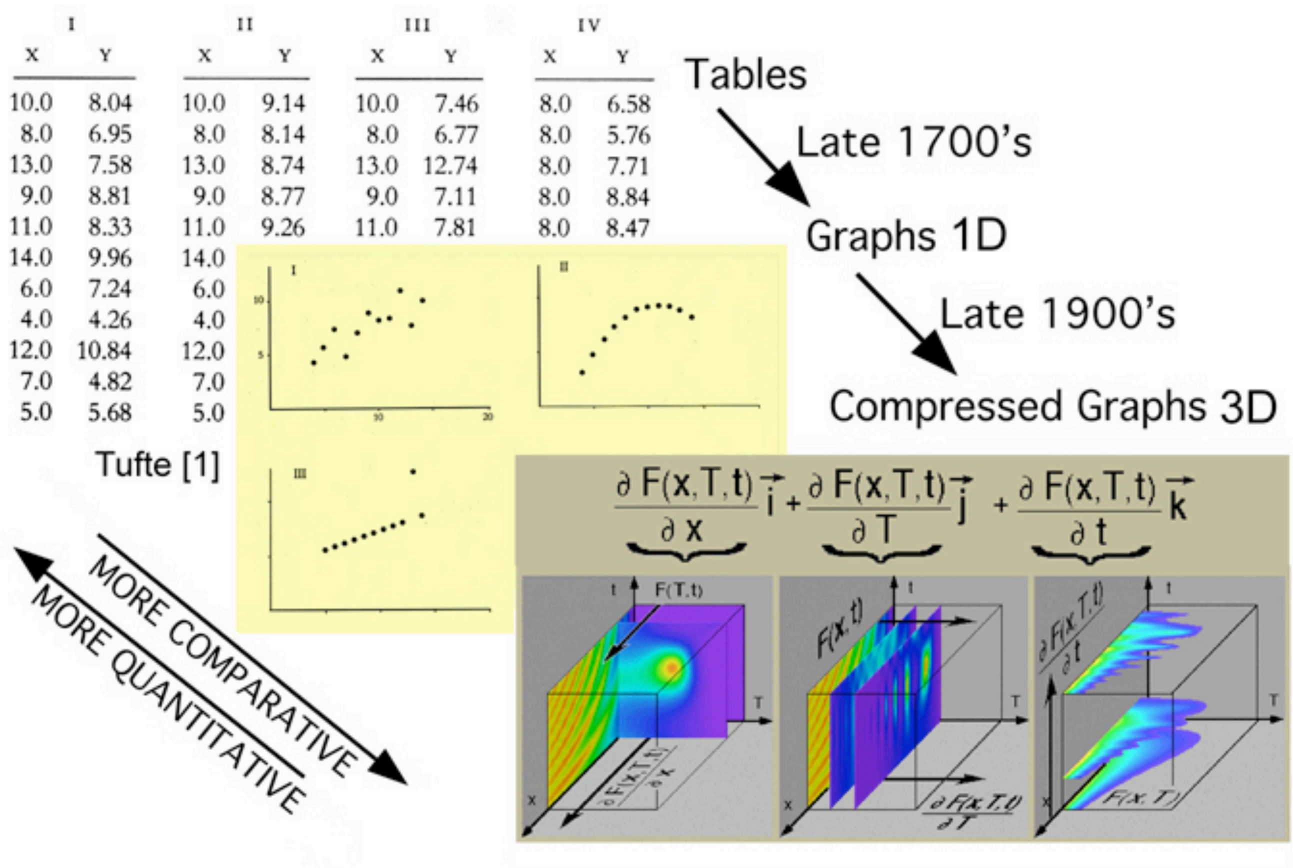


Figure 10. Chronology of envisioning scientific information

Richard Friedhoff [2] also points out that this relationship between functions and visual patterns is fundamental to the way we create our functions to begin with. For example J.W.Gibbs, J.C. Maxwell and others have made comments to support this claim but only mathematical script (equations) or simple graphs can be used to communicate these ideas to others. Hence the original thought that created these functions is not communicated although this very initial process is what inspired Maxwell to create clay and plaster models of Gibb's "visual mental model" of the thermodynanmic properties of water. This point is discussed by Tom West [3] in brief article on the "Return to Visual Thinking".

Because we think before we draw a pattern that represents a function, this cognitive process has become unidirectional. This cognitive connection between thinking and then drawing is inherently implicit to the point that we do not realize how unidirectional this process has become until we, for example, return to Fig. 7 and select a plane where the computer draws either the x-t or t-T planes and we realize that the reciprocal could also be true: patterns can come before thinking about the functions, hence, we have the opportunity to discover functions from patterns. This cognitive mechanism between functions and patterns does not presuppose the order. Which comes first is an irrelevant question. As investigators and instructors we can use both: for presentation (thought first then pattern) and for investigation (pattern first and then thought). If this thought-then-pattern cognitive mechanism were truly unidirectional then computer graphics could only be used for presentation.

Returning to Fig. 7 we see that the original 90,000 curves can now be shown in yet two more planes, hence, we have an additional 180,000 curves making a total of 270,000 curves to think about. Of course present graphic technology is not limited to m=300 and n=300. Hence we can work with even larger and more massive data sets.

It first "appears" that we can only use this method for a single scalar function. It would not be possible to observe more then one color gradient (scalar-function) at a time if we are confined to observe how color (function) changes in a moving plane. But we can invoke other graphic features that can allow us to include more then one function into the same parametric space. Hence we have the opportunity to visually explore multiple functional relationships if each function has the same basis (parameter space). But before we generalize our method to more then one function, we provide some simple, hopefully instructive, examples of investigating single functions in parametric space using CVDC method.

One closing comment before we proceed to specific examples. Keep in mind that as we go through these examples, the usefulness of this cognitive visual approach is not limited to a particular application. Hence we believe this to be a general methodology.

Two Case Studies Using the CVDC Method: Parametric Design:

1. Choose a Squadron of Pilots: Experimental Data
2. Material Property Selection: Analytic Function.

Send comments to: rkriz@vt.edu
Ronald D. Kriz, Short Bio
Engineering Science and Mechanics
College of Engineering
Virginia Tech
Blacksburg, Virginia 24061
Revised July 10, 2006

http://www.sv.vt.edu/classes/ESM4714/methods/CogVizCmp.html
